Theory of Music – Jonathan Dimond

Golden Section

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INTRODUCTION

Golden section, Golden ratio, Golden proportion, *sectio aurea* (Latin), divine proportion, divine section – these are all similes for the same phenomenon.

In its simplest expression as a line segmented into two parts, Golden section (GS) is the point at which you divide the line such that the ratio of the length of the first part to the second is the same as the second to the whole.

l.e. a:b = a+b:a

Or b:a = a:a+b



a+b is to a as a is to b

http://en.wikipedia.org/wiki/Golden_ratio

The resulting figure is represented by the Greek letter Phi - φ . Numerically speaking, Phi – like Pi, is an irrational number. That is, the Golden ratio creates a number with a never-ending series of decimal places.

 $(1 + \sqrt{5}) / 2 = 1.618033987...$

We tend to round this number to three decimal places. The way we work with the numerical expression of this number is by multiplying by 1.618 (to produce a larger number in GS) or 0.618 (to produce a smaller one). Also, we tend to refer to the "negative GS" as the remainder from the unit of 1 once 0.618 is taken away. I.e., 1-0.618=0.382. This is sometimes symbolized by the capital letter version of phi.

Since about 400 B.C. mathematicians, artists, biologists, architects, and astronomers have been attracted to the proportions that Phi represents, have pondered its universal appearance in the structure and organization of almost all living and natural things, and consciously employed it in the design of their art and work. We even have evidence of its use in the pyramids of ancient Egypt as far back as 2500 B.C. (Madden, p. 1.) Golden section's proportions seem to create an aesthetic which is naturally appealing.

Golden proportion can be applied to create geometrical shapes. The following method is used to construct a Golden rectangle:

- 1. Draw a square, 1x1 unit wide/long.
- 2. From the midpoint of one side draw a line to the opposite corner.
- 3. Continue that line as a radius to sweep an arc beyond the square. This defines the new rectangle's length.

The ratio of the rectangle's added length to the square's is the same as the square's to the whole of the rectangle's length.



Measure the business cards and credit cards in your wallet. The international standard size for credit cards, which is widely used for business cards also is 85.60 × 53.98 mm (3.370 × 2.125 in). Divide the width by the length and you will always come up with the result around 0.618. This is a common ratio which is found even in the more unusual sizes found in the USA, Japan and Italy. Why? It is a pleasing proportion and fits neatly into our hand.

The earliest major record of GS and the Golden rectangle surviving today dates from around 490 B.C. with the Greek Parthenon in Athens. (The Greek mathematician Pythagorus and his followers are said to have developed GS theories from around the same time.) As seen below, the façade fits into a Golden rectangle, and inner Golden rectangles dictate the proportions of the structure including the slope of the roof.



A more contemporary example of GS and Golden rectangles is in the design of page sizes and text layout in books. In the 16th Century, the following format for a 2-page spread emerged:



A scholar named Tschichold found the ratio 34:21 to be prevalent, which is 0.617, and this proportion organizes not only the page size but the placement and proportion of the margins. Many books produced between 1550 and 1770 show these proportions exactly, to within half a millimetre.

Other "Golden" proportioned geometric shapes include triangles, pentagons and pyramids. <u>http://en.wikipedia.org/wiki/Golden_ratio</u>



Most interesting is Golden sections role in creating natural spirals.



Spirals are naturally occurring in nature, and we find them throughout the universe and within natural living things. [See drawings and photos in Doczi]

Musical Harmony

In musical harmony, and the development of temperament (tuning systems), we discovered early on that it was pleasurable to listen to strings tuned in relations of small integers – i.e. using the numbers 1, 2, 3, 4... The following lists ratios and their interval equivalents:

- 1:1 unison
- 1:2 octave (*diapason*)
- 2:3 perfect fifth (*diapente*)
- 3:4 perfect fourth (*diatessaron*)
- 4:5 major third (*ditone*)

Expressed as a decimal, the perfect fifth is 2/3 = 0.666... which approximates GS. No wonder it is the next most consonant-sounding interval after the unison and octave, and is such a common place to modulate. Furthermore, we find these intervals early on in the natural harmonic series.

For further reading on the historical usage of these intervals in theory and composition, refer to translations of the book "Micrologus" by Guido of Arezzo. Guido was a Frenchman living in Italy as a monk, and established the fixed-*do* system of solfege.

Fibonacci Series

Closely related to GS is the Fibonacci series. Named after the 12th Century Italian mathematician of the same name, the Fibonacci series is a sequence of numbers generated by adding together the prior two numbers in the sequence. (Evidence of the series also exists in Indian literature and music of around 300 B.C.) <u>http://en.wikipedia.org/wiki/Fibonacci_number</u>

The first 14 Fibonacci numbers are:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377

The series was first used to describe proliferation of mating rabbits, bees, and in the design of Indian rituals involving prosody (chants/utterances concerned with intonation, rhythm and focus in speech.)

In terms of Golden Section, the result of dividing any two adjacent numbers in the Fibonacci series is that we approximate Phi. The series tends towards true Phi the further along the series we progress.

Consider the piano keyboard. We have two groups of black notes – a group of 2 and a group of 3, totalling 5. In one octave, C to C, we have 8 white notes, giving the grand total of 13 notes. 2, 3, 5, 8, and 13 are all from the Fibonacci series.

Try this math game.

- 1. Choose a number
- 2. Square each of its digits
- 3. Add the resulting new numbers together to create a new number
- 4. Repeat the process (back to step 2)

The sequence will either descend to 1 (the first Fibonacci number) or loop around to 89 – the 11th Fibonacci number. (Spencer, 2000, p.184.)

BIBLIOGRAPHY & DISCOGRAPHY:

- Doczi, Gyorgy. The Power of Limits: Proportional Harmonies in Nature, Art, and Architecture. Shambhala, Boston, 1994.
- Lendvai, Erno. Bela Bartok : an analysis of his music. Kahn & Averill, London, 1971. [780.924 BAR:L]
- Spencer, Adam. Book of Numbers. Penguin Australia, 2000.
- Madden, Charles. *Fib and Phi in Music The Golden Proportion in Musical Form.* Salt Lake City: High Art Press, 2005. [781.2 MAD]